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# Vibration modes of giant gravitons in the background of dilatonic D-branes

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## Abstract

We consider the perturbation of giant gravitons in the background of dilatonic D-branes whose geometry is not of a conventional form of  $\text{AdS}_m \times \text{S}^n$ . We use the quadratic approximation to the brane action to investigate their vibrations around the equilibrium configuration. We found the normal modes of small vibrations of giant gravitons and these vibrations are turned out to be stable.

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Recently stable extended brane configurations in some string theory background, called giant gravitons, attracted interests in connection with the stringy exclusion principle. Myers [1] found that certain D-branes coupled to RR potentials can expand into higher dimensional branes. McGreevy, Susskind and Toumbas [2] have shown that a massless particle with angular momentum on the  $S$  part of  $\text{AdS}_m \times S^n$  spacetime blows up into a spherical brane of dimensionality  $n-2$ . Its radius increases with increasing angular momentum. The maximum radius of the blown-up brane is equal to the radius of the sphere that contains it since the angular momentum is bounded by the radius of  $S^n$ . This is a realization of the stringy exclusion principle [3] through the AdS/CFT correspondence [4]. Later it was shown that the same mechanism can be applied to spherical branes on the AdS part [5,6]. However, they can grow arbitrarily large since there is no upper bound on the angular momentum. To solve this puzzle, instanton solutions describing the tunneling between the giant gravitons on the AdS part and on the  $S$  part were introduced [5,7]. A magnetic analogue of the Myers effect was investigated by Das, Trivedi and Vaidya [8]. They suggested that the blowing up of gravitons into branes are also possible on some backgrounds other than  $\text{AdS}_m \times S^n$  spacetime

Perturbations of the giant gravitons around their equilibrium configuration were studied by Das, Jevicki and Mathur [9]. Using quadratic approximation to the action, they computed the natural frequency of the normal mode for giant gravitons in  $\text{AdS}_m \times S^n$  spacetime for both cases when gravitons are extended in AdS space and they are extended on the sphere. Frequencies are related to the curvature scale of the background and are independent of the radius of the brane itself. These modes are believed to play an important role in the study of black holes within the string theory context. We cannot explain the microscopic properties of a black hole if it is regarded as a point singularity. But in string theory, a pointlike singularity can be replaced by an extended brane, and vibration modes of the brane can be used to study the microscopic entropy [10] and Hawking radiation [11].

In this paper, we will study the vibration mode of giant gravitons in the near-horizon geometry of dilatonic D-brane background. As a concrete example, we will consider the case

when a probe  $p$ -brane wraps the transverse direction of  $S^{p+2}$  in the near-horizon geometry of  $D(6-p)$  branes [8]. Since the background geometry is not exactly of the form  $AdS_m \times S^n$ , it would be very interesting to study the fluctuation analysis around this configuration.

Consider a probe  $Dp$ -brane moving in the near-horizon geometry of extremal  $N$   $D(6-p)$ -branes. The background metric is given by

$$ds^2 = g_{tt}dt^2 + \sum_{k=1}^{6-p} g_{kk}(dx_k)^2 + g_{rr}dr^2 + f(r)r^2d\Omega_{p+2}^2, \quad (1)$$

where

$$g_{tt} = g_{kk} = \left(\frac{r}{R}\right)^{\frac{(p+1)}{2}}, \quad g_{rr} = f(r) = \left(\frac{R}{r}\right)^{\frac{(p+1)}{2}}, \quad (2)$$

and  $R$  can be expressed in terms of  $N$  and the tension of a probe  $Dp$ -brane as  $R^{p+1} = N/T_p V_p$ , where  $V_p = 2\pi^{\frac{p+1}{2}}/\Gamma(\frac{p+1}{2})$  is the volume of unit  $S^p$ .  $d\Omega_{p+2}^2$  can be parametrized as

$$d\Omega_{p+2}^2 = \frac{1}{1-\rho^2}d\rho^2 + (1-\rho^2)d\phi^2 + d\Omega_p^2, \quad (3)$$

with

$$d\Omega_p^2 = d\theta_1^2 + \sin^2\theta_1(d\theta_2^2 + \sin^2\theta_2(\cdots + \sin^2\theta_{p-1})d\theta_p^2). \quad (4)$$

Also we have the dilaton ( $\Phi$ ) and RR potential ( $A^{p+1}$ ) given by

$$e^\Phi = \left(\frac{R}{r}\right)^{\frac{(p-3)(p+1)}{4}}, \quad (5)$$

$$A_{\phi\theta_1\cdots\theta_p}^{p+1} = R^{p+1}\rho^{p+1}\epsilon_{\theta_1\cdots\theta_p}, \quad (6)$$

where  $\epsilon_{\theta_1\cdots\theta_p}$  is the volume form of the unit  $p$ -sphere. In the case of  $r = R$ ,  $\Phi = 0$ . For  $0 < r < R$ , we have a non-zero dilaton. In this sense, Eq.(1) is called the dilatonic  $D$ -brane background.

We consider an equilibrium configuration in which a probe  $Dp$ -brane wraps the  $S^p$ . The action for this case, ignoring the fermions, is given by

$$\begin{aligned} S &= S_{DBI} + S_{CS} \\ &= -T_p \int d^{p+1}\xi e^{-\Phi} \sqrt{-\det \hat{G}_{\alpha\beta}} + T_p \int d^{p+1}\xi \hat{A}^{p+1}, \end{aligned} \quad (7)$$

where  $\hat{G}_{\alpha\beta}$  and  $\hat{A}^{p+1}$  are pullbacks of the metric and the RR  $(p+1)$ -form potential, respectively

$$\hat{G}_{\alpha\beta} = \frac{\partial x^M}{\partial \xi^\alpha} \frac{\partial x^M}{\partial \xi^\beta} g_{MN}, \quad (8)$$

$$\hat{A}_{\xi_0 \xi_1 \dots \xi_p}^{(p+1)} = A_{M_1 \dots M_p}^{(p+1)} \frac{\partial x^{M_1}}{\partial \xi_0} \frac{\partial x^{M_2}}{\partial \xi_1} \frac{\partial x^{M_{p+1}}}{\partial \xi_p}. \quad (9)$$

If we choose a static gauge : the time parameter of the worldvolume  $\xi^0 = t$  ; the  $p$  angular parameters  $\xi_i$  are set to be the angles on  $S^p$ ,  $\xi_i = \theta_i$ , then the dynamical variables are given by  $r(t, \theta_i)$ ,  $x_k(t, \theta_i)$ ,  $\rho(t, \theta_i)$  and  $\phi(t, \theta_i)$ . We choose an equilibrium configuration such that these quantities do not depend on  $\theta_i$  so that there are no brane oscillations. Since there exist translational symmetries along  $x^i$ , the corresponding momenta are conserved. We will study the motion whose conserved momenta are identically zero. Then, the dynamical variables take the form of  $r(t)$ ,  $\rho(t)$  and  $\phi(t)$ . With an appropriate choice of gauge, we get the full probe brane action as

$$S = -T_p V_p \int dt e^{-\Phi} (f(r) \rho^2 r^2)^{p/2} \sqrt{g_{tt} - g_{rr} \dot{r}^2 - g_{\rho\rho} \dot{\rho}^2 - g_{\phi\phi} \dot{\phi}^2} + N \int dt \rho^{p+1} \dot{\phi}, \quad (10)$$

where dot denotes derivative with respect to  $t$  and  $V_p$  stands for the volume of unit  $p$ -sphere. Its conjugate momenta and Hamiltonian are:

$$P_r = \frac{\partial L}{\partial \dot{r}} = \frac{T_p V_p e^{-\Phi}}{\sqrt{g_{tt} - g_{rr} \dot{r}^2 - g_{\rho\rho} \dot{\rho}^2 - g_{\phi\phi} \dot{\phi}^2}} (f \rho^2 r^2)^{p/2} g_{rr} \dot{r}, \quad (11)$$

$$P_\rho = \frac{\partial L}{\partial \dot{\rho}} = \frac{T_p V_p e^{-\Phi}}{\sqrt{g_{tt} - g_{rr} \dot{r}^2 - g_{\rho\rho} \dot{\rho}^2 - g_{\phi\phi} \dot{\phi}^2}} (f \rho^2 r^2)^{p/2} g_{\rho\rho} \dot{\rho}, \quad (12)$$

$$P_\phi = \frac{\partial L}{\partial \dot{\phi}} = \frac{T_p V_p e^{-\Phi}}{\sqrt{g_{tt} - g_{rr} \dot{r}^2 - g_{\rho\rho} \dot{\rho}^2 - g_{\phi\phi} \dot{\phi}^2}} (f \rho^2 r^2)^{p/2} g_{\phi\phi} \dot{\phi} + N \rho^{p+1}, \quad (13)$$

$$\begin{aligned} H &= P_r \dot{r} + P_\rho \dot{\rho} + P_\phi \dot{\phi} - L \\ &= \sqrt{g_{tt}} \left[ (T_p V_p e^{-\Phi})^2 (f(r) \rho^2 r^2)^p + \frac{P_r^2}{g_{rr}} + \frac{P_\rho^2}{g_{\rho\rho}} + \frac{(P_\phi - N \rho^{p+1})^2}{g_{\phi\phi}} \right]^{1/2}. \end{aligned} \quad (14)$$

Note that if we impose a condition

$$T_p V_p e^{-\Phi} (f(r) r^2)^{\frac{p+1}{2}} = N, \quad (15)$$

and use  $g_{\phi\phi} = f(r)r^2(1 - \rho^2)$ , we can combine the first and last terms within the square brackets of Eq. (14). Then the Hamiltonian can be expressed as

$$H = \sqrt{g_{tt}} \left[ \frac{P_\phi^2}{f(r)r^2} + \frac{P_r^2}{g_{rr}} + \frac{P_\rho^2}{g_{\rho\rho}} + \frac{(\rho P_\phi - N\rho^p)^2}{g_{\phi\phi}} \right]^{1/2}. \quad (16)$$

Since the Lagrangian does not depend on  $\phi$ ,  $P_\phi$  is a constant of motion. For a given  $P_\phi$ , the lowest energy configuration satisfies  $P_\rho = 0$  for all time because  $\rho$  does not appear in the first two terms. Here we find a relation between the equilibrium value of  $\rho_0$  and  $P_\phi$

$$P_\phi = N\rho_0^{p-1}, \quad (17)$$

which is obviously independent of the radial coordinate  $r$ . The corresponding Hamiltonian is

$$H = \sqrt{g_{tt}} \left[ \frac{P_\phi^2}{f(r)r^2} + \frac{P_r^2}{g_{rr}} \right]^{1/2}. \quad (18)$$

Actually this is the Hamiltonian of a massless particle with angular momentum  $P_\phi$  on  $S^{p+2}$  sphere. Unlike the usual massless particle, the angular momentum of a probe brane is bounded because of  $0 \leq \rho \leq 1$ . A possible maximum value of its angular momentum is  $N$ . This reminds us the stringy exclusion principle. Eq.(15) is an important condition for the brane to behave like a massless particle.

So far we reviewed how the configuration of giant graviton appears in the near-horizon background of dilatonic D-brane. Now we consider the small vibration of giant graviton around the stable equilibrium configuration. In general, the brane can move in any direction. We can consider the brane motion along any of the  $r$ ,  $\rho$  and  $\phi$ . If one considers the case when the brane moves along the  $r$  direction, one can get the cosmological model known as the mirage cosmology [12]. The motion of the probe (or universe) brane in ambient space induces cosmological expansion (or contraction). And if we consider the brane motion along the  $\rho$  and  $\phi$  directions, we get the giant graviton picture as in Ref. [8]. Since we are interested in the perturbation from stable configuration of the giant graviton, we neglect the motion of the probe brane along the transverse direction ( $r$ ) of the background  $D(6-p)$ -branes. So

we set  $\dot{r} = 0$ , i.e.  $r = r_0 = \text{constant}$ . We find the angular velocity of this configuration from Eqs. (13) and (17)

$$\dot{\phi} \equiv \omega_0 = \pm \frac{1}{f_0 r_0} \quad (19)$$

with  $f_0 = f(r_0) = (r_0/R)^{(p+1)/2}$ .

A small vibration of the brane can be described by defining spacetime coordinates  $(\rho, \phi, x_k)$  as a function of the worldsheet coordinates  $\xi_0, \xi_1, \dots, \xi_p$ . In the static gauge, where

$$\xi^0 = t = \tau, \quad \xi_i = \theta_i, \quad i = 1, \dots, p, \quad (20)$$

perturbation of the remaining coordinates can be written as

$$\rho = \rho_0 + \epsilon \delta\rho(\tau, \xi_1, \dots, \xi_p), \quad (21)$$

$$\phi = \omega_0 \tau + \epsilon \delta\phi(\tau, \xi_1, \dots, \xi_p), \quad (22)$$

$$x_k = \epsilon \delta x_k(\tau, \xi_1, \dots, \xi_p), \quad k = 1, \dots, 6-p. \quad (23)$$

First we expand the action (7) to linear-order in  $\epsilon$

$$\begin{aligned} S(\epsilon) = & -\epsilon T_p \int d\tau d^p \xi \sqrt{g_\xi} e^{-\Phi_0} (f_0 r_0^2)^{\frac{p}{2}} \rho_0^{p-1} \\ & \times \left[ \frac{(p+1)f_0 r_0^2 \omega_0^2 \rho_0^2 + p(1/f - f_0 r_0^2 \omega_0^2)}{\sqrt{1/f - f_0 r_0^2 (1 - \rho_0^2) \omega_0^2}} \delta\rho - \frac{(1 - \rho_0^2) f_0 r_0^2 \omega_0 \rho_0}{\sqrt{1/f - f_0 r_0^2 (1 - \rho_0^2) \omega_0^2}} \delta\dot{\phi} \right] \\ & + \epsilon N \int d\tau \rho_0^p [(p+1)\omega_0 \delta\rho + \rho_0 \delta\dot{\phi}]. \end{aligned} \quad (24)$$

Using Eq.(15), we have

$$\begin{aligned} S(\epsilon) = & -\epsilon N \rho_0^{p-1} \int d\tau \left[ \left\{ \frac{1}{\sqrt{f_0 r_0^2}} \frac{(p+1)f_0 r_0^2 \omega_0^2 \rho_0^2 + p(1/f - f_0 r_0^2 \omega_0^2)}{\sqrt{1/f - f_0 r_0^2 (1 - \rho_0^2) \omega_0^2}} - (p+1)\rho_0 \omega_0 \right\} \delta\rho \right. \\ & \left. + \left\{ \frac{1}{\sqrt{f_0 r_0^2}} \frac{-(1 - \rho_0^2) f_0 r_0^2 \omega_0 \rho_0}{\sqrt{1/f - f_0 r_0^2 (1 - \rho_0^2) \omega_0^2}} - \rho_0^2 \right\} \delta\dot{\phi} \right]. \end{aligned} \quad (25)$$

Clearly the coefficient of  $\delta\rho$  vanishes if one uses  $\omega_0 = \pm \frac{1}{f_0 r_0}$ . Also the coefficient of  $\delta\dot{\phi}$  is a constant and thus this term does not contribute to the variation of the action upon the integration over  $\tau$ .

On the other hand, the second order term in  $\epsilon$  is calculated as

$$\begin{aligned}
S(\epsilon^2) &= \epsilon^2 \frac{N}{T_p} \omega_0 \rho_0^{p-1} \int d\tau d^p \xi \sqrt{g_\xi} \\
&\times \left[ \frac{1}{2} \frac{1}{\omega_0^2 (1 - \rho_0^2)} (\delta \dot{\rho})^2 - \frac{1}{2} \frac{1}{(1 - \rho_0^2)} \frac{\partial \delta \rho}{\partial \xi_i} \frac{\partial \delta \rho}{\partial \xi_j} g^{\xi_i \xi_j} \right. \\
&+ \frac{1}{2} \frac{1 - \rho_0^2}{\omega_0^2 \rho_0^2} (\delta \dot{\rho})^2 - \frac{1}{2} (1 - \rho_0^2) \frac{\partial \delta \phi}{\partial \xi_i} \frac{\partial \delta \phi}{\partial \xi_j} g^{\xi_i \xi_j} \\
&+ \frac{p-1}{\omega_0 \rho_0} \delta \rho \delta \dot{\rho} \\
&+ \left. \frac{1}{2} \sum_{k=1}^{6-p} (\delta \dot{x}_k)^2 - \frac{1}{2} \frac{1}{f_0 r_0^2} \sum_{k=1}^{6-p} \frac{\partial \delta x_k}{\partial \xi_i} \frac{\partial \delta x_k}{\partial \xi_j} g^{\xi_i \xi_j} \right]. \tag{26}
\end{aligned}$$

The equations of motion are

$$\frac{1}{\omega_0^2 (1 - \rho_0^2)} \frac{\partial^2 \delta \rho}{\partial \tau^2} - \frac{1}{1 - \rho_0^2} \frac{\partial}{\partial \xi_i} \left( \frac{\partial \delta \rho}{\partial \xi_j} g^{\xi_i \xi_j} \right) - \frac{p-1}{\omega_0 \rho_0} \frac{\partial \delta \phi}{\partial \tau} = 0, \tag{27}$$

$$\frac{1 - \rho_0^2}{\omega_0^2 \rho_0^2} \frac{\partial^2 \delta \phi}{\partial \tau^2} - (1 - \rho_0^2) \frac{\partial}{\partial \xi_i} \left( \frac{\partial \delta \phi}{\partial \xi_j} g^{\xi_i \xi_j} \right) + \frac{p-1}{\omega_0 \rho_0} \frac{\partial \delta \rho}{\partial \tau} = 0, \tag{28}$$

$$\frac{\partial^2 \delta x_k}{\partial \tau^2} - \frac{1}{f_0^2 r_0^2} \frac{\partial}{\partial \xi_i} \left( \frac{\partial \delta x_k}{\partial \xi_j} g^{\xi_i \xi_j} \right) = 0. \tag{29}$$

We observe that  $\delta x_k$  perturbations are decoupled from  $\delta \rho$  and  $\delta \phi$ . Let us Introduce the spherical harmonics  $Y_l$  on  $S^p$ ,

$$g^{\xi_i \xi_j} \frac{\partial}{\partial \xi_i} \frac{\partial}{\partial \xi_j} Y_l(\xi_1, \dots, \xi_p) = -Q_l Y_l(\xi_1, \dots, \xi_p), \tag{30}$$

where  $Q_l$  is the eigenvalue of the Laplace operator on unit  $p$  sphere. Possible values of  $Q_l$  are  $l(l + p - 1)$  with  $l = 0, 1, 2, \dots$ . Choosing the harmonic oscillation, perturbations can be expressed as

$$\delta \rho(\tau, \xi_1, \dots, \xi_p) = \tilde{\delta} \rho e^{-i\omega\tau} Y_l(\xi_1, \dots, \xi_p), \tag{31}$$

$$\delta \phi(\tau, \xi_1, \dots, \xi_p) = \tilde{\delta} \phi e^{-i\omega\tau} Y_l(\xi_1, \dots, \xi_p), \tag{32}$$

$$\delta x_k(\tau, \xi_1, \dots, \xi_p) = \tilde{\delta} x_k e^{-i\omega\tau} Y_l(\xi_1, \dots, \xi_p). \tag{33}$$

From Eq. (29), we find the natural frequency for  $\delta x_k$  perturbations as

$$\omega_x^2 = \frac{Q_l}{(f_0 r_0)^2} = Q_l \omega_0^2 \rightarrow \omega_x = \pm \sqrt{Q_l} \omega_0. \tag{34}$$

And  $\delta\rho$  and  $\delta\phi$  perturbations are coupled and their normal frequencies are determined by the matrix equation

$$\begin{pmatrix} \frac{1}{(1-\rho_0^2)}(-\frac{\omega^2}{\omega_0^2} + Q_l) & i\omega(p-1)\frac{1}{\omega_0\rho_0} \\ -i\omega(p-1)\frac{1}{\omega_0\rho_0} & \frac{(1-\rho_0^2)}{\rho_0^2}(-\frac{\omega^2}{\omega_0^2} + \rho_0^2 Q_l) \end{pmatrix} \begin{pmatrix} \tilde{\delta}\rho \\ \tilde{\delta}\phi \end{pmatrix} = 0. \quad (35)$$

This gives the frequencies of two modes ( $\pm$ )

$$\omega_{\pm} = \frac{1}{2(f_0 r_0)^2} \left[ (1 + \rho_0^2)Q_l + (p-1)^2 \pm \sqrt{(p-1)^4 + 2(p-1)^2(1 + \rho_0^2)Q_l + Q_l^2(1 - \rho_0^2)^2} \right]. \quad (36)$$

These modes oscillate with real and positive  $\omega_{\pm}$ , so their vibration is stable for any size of the probe brane. To make a connection with  $\text{AdS}_m \times \text{S}^n$ , we can choose the possible maximum value of  $\rho_0$  as  $\rho_0 = 1$ . Then the natural frequencies for this case are

$$\omega_{\pm} = \frac{1}{(f_0 r_0)^2} \left[ Q_l + \frac{(p-1)^2}{2} \pm (p-1) \sqrt{Q_l + \frac{(p-1)^2}{4}} \right]. \quad (37)$$

This agrees with the result of the case whose background geometry is of the form  $\text{AdS}_m \times \text{S}^n$  (see, Eq.(4.17) of [9] and Eq.(73) of [13]). Inserting  $Q_l = l(l+p-1)$ , we obtain  $\omega_+ = (1/f_0 r_0)(l+p-1)$  and  $\omega_- = (1/f_0 r_0)l$ . Since  $l$  is integer, their motion is periodic. Here we would like to emphasize that Eq. (15) is the crucial condition in our calculation. It has been discussed in [8] that we can draw the giant graviton picture even in non-supersymmetry preserving backgrounds whenever this condition is met.

It is worthwhile commenting that the case considered in this paper is somewhat different from that in  $\text{AdS}_m \times \text{S}^n$  space. Extended brane configurations in  $\text{AdS}_m \times \text{S}^n$  space, relevant for the giant graviton picture, sit at definite value of  $r$  both for branes extended in AdS subspace and for branes extended on the sphere in which case the brane sits at the origin of the AdS space. In our calculation we did not consider the brane motion along the transverse direction ( $r$ ) in the background  $D(6-p)$ -branes. Interestingly it is known that the brane motion in this direction induces a cosmological evolution on the universe brane, called the mirage cosmology [12,14]. Recently it has been studied that the motion of giant graviton is related to the closed universe of mirage cosmology [14]. Hence the motion of a probe

*p*-brane along the *r*-direction is expected to induce an interesting cosmological evolution in the background whose geometry is not  $\text{AdS}_m \times \text{S}^n$  space. If we remove the restriction  $\dot{r} = 0$ , then the only change might be the existence of the additional vibrational modes along the *r* direction which do not mix with the modes considered here. It would be interesting to study the vibration modes without the restriction  $\dot{r} = 0$ .

In summary, we found the normal modes of small vibrations of giant gravitons in the background of dilatonic D-branes whose geometry is not of the form  $\text{AdS}_m \times \text{S}^n$  and these vibrations are stable.

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